Financial Econometrics
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TP 8 – Solution

Bootstrap and Asymptotic Confidence Intervals

The Delta Method

From the property, we have:

$$\Sigma = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^4 \end{pmatrix}.$$

Then, using the hint we obtain:

$$\frac{\partial g}{\partial \theta} = \begin{pmatrix} \frac{1}{\sqrt{\sigma^2}} & -\frac{1}{2}\mu\sigma^{-3} \end{pmatrix}.$$

Hence:

$$\frac{\partial g}{\partial \theta}\Sigma\frac{\partial g'}{\partial \theta} = 1 + \frac{1}{4}\left(\frac{\mu}{\sigma}\right)^2.$$

Denoting by $z_{\alpha/2}$ and $z_{1-\alpha/2}$ the $\alpha/2$ and $1-\alpha/2$ quantiles of $\mathcal{N}(0, 1 + \frac{1}{4}(\frac{\hat{\mu}}{\hat{\sigma}})^2)$, and using the delta method, we can write:

$$\mathrm{P}\Big(z_{\alpha/2} \leq \sqrt{T}(g(\hat{\theta}) - g(\theta)) \leq z_{1-\alpha/2}\Big) = 1 - \alpha.$$

Rearranging:

$$\mathrm{P}\Big(g(\hat{\theta}) - \frac{z_{1-\alpha/2}}{\sqrt{T}} \leq g(\theta) \leq g(\hat{\theta}) - \frac{z_{\alpha/2}}{\sqrt{T}}\Big) = 1 - \alpha,$$

which gives the asymptotic confidence interval for SR.